

Chapter 7:

DC Motors and Transmissions

Electric motors are one of the most common types of [actuators](#) found in robotics. Using them effectively will allow your robot to take action based on the direction given by its sensors and programming. Although there are [many types of electric motors](#), this section will focus on the mechanics, mathematics, and proper use of DC permanent magnet, brushed motors. Commonly referred to as PMDC motors, they are a popular choice due to their small size and cost, and the fact that unlike many DC motors, powering them is as simple as connecting a constant voltage to the motor.

7.1: Basic Definitions and Concepts

A motor is an imperfect [transducer](#). Motors are used to convert electrical power into mechanical power, in this case, a [torque](#) applied to the motor's output shaft. However, motors will also inevitably transform some of the given electrical power into thermal energy, heating up the motor instead of producing useful work.

Both power input and output can be measured in any unit of power, typically Watts. Power is the rate at which work is done. In mechanical terms, high power implies that not only is a load being moved with a great deal of speed, but continuing to move the load demands a great deal of force of a system. In terms of a rotating motor, power is calculated using the formula:

$$P = T * \omega * \text{Scale Factor}$$

For more on relationships between power, torque, and speed, [return to transmissions](#).



Figure 1: A number of sizes of Permanent Magnet DC motors.

Where T is the torque being output by the motor, ω is the angular velocity of the output, and the Scale Factor is used to correct for varied units. Some common scale factors are listed in the table [here](#).

A key point to remember is that torque and velocity are equally important in the power formula. You could have X torque and Y speed, or Y torque and X speed, and get the exact same amount of power. If you ever had zero speed or zero torque, there would be no power.

Electrical power, also measured in Watts, is calculated using the formula $P=IV$, where I is electrical current in Amperes, and V is the motor's voltage. For example, a CIM motor that is drawing 40 amps while connected to a 12 volt supply is using 480 Watts of electrical power.

Motors are typically rated according to the mechanical power that they can output. If at the same time as this CIM motor draws 480 Watts of electrical power, it is spinning at 3800RPM and outputting 6.15 in*lbs of torque, it is outputting 275 Watts of power, based on the conversion factor for in*lbs and RPM to Watts found [here](#).

Obviously a significant amount of power was lost in this process. Dividing power output by power input produces an important value for all motors called efficiency. Typically presented as a percent, efficiency tells you how much of the electrical power being input to the motor is actually being converted to useful mechanical power. The remaining power, in this case 205 Watts, is wasted as heat.

7.2: Internal Workings of a PMDC Motor

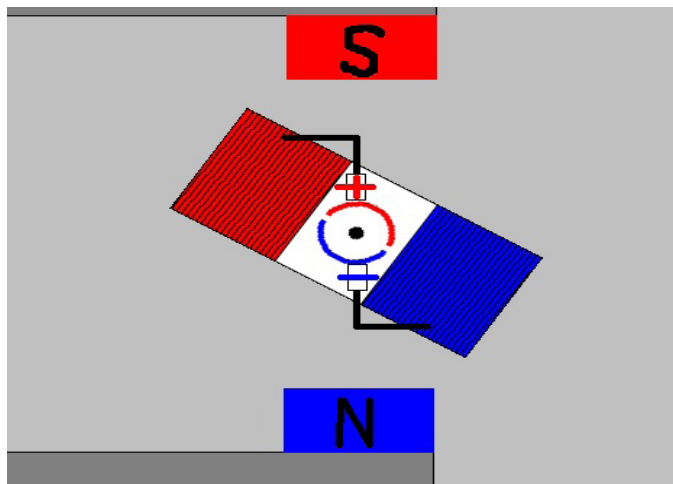
All electric motors use electromagnets to generate forces within the motor, creating rotation. In a PMDC motor, these electromagnets react with a pair of permanent magnets, created from naturally ferromagnetic materials, treated to permanently maintain their magnetic field.

A Permanent Magnet DC motor is divided into two main parts, the stationary stator, and the rotating armature. The permanent magnets are placed on opposite ends of the stator, with opposite poles facing inwards. In between these magnets lies the armature, mounted on bearings and connected to the output shaft of the motor. The armature consists of a metal core, typically iron, which will readily take on magnetic properties. The armature is wrapped in one or more copper coils. This combination serves as a powerful electromagnet when current is passed through the coils.

Imagine for a second energizing this electromagnet. The armature will take on the properties of a permanent magnet. Forces will be generated at each end of the armature, caused by attraction to one end of the stator, and opposing the other. This creates a couple moment, forcing the armature to turn. However, as can be seen in the slideshow, the armature will quickly reach a stable equilibrium position, where the forces do not generate a moment, and any disturbance in either direction will result in the armature being pulled back to this stationary position.

This problem is solved by a set of components called the Commutator and brushes. The commutator is typically represented as a pair of half-circle copper rings, with a small gap or piece of insulation between them. The brushes are metal strips or blocks which run along the outside surface of the commutator. The commutator rotates with the armature, and the brushes remain fixed to the stator. The coils of the armature are electrically connected to the commutator rings. Current is passed through the brushes, which is then transmitted to the armature through their contact with the commutator.

However, each time the commutator makes a half-rotation, the rings switch which brush they were in contact with (see this animated [here](#)). This has the effect of flipping the current flow in the armature. Inverting the direction of current also inverts the magnetic field generated by the armature. In the motor, the commutator is arranged such that this flip occurs just as the motor reaches the equilibrium position described above. But now, flipping the current will pull the armature through another 180 degrees of rotation. This cycle continues every half rotation, allowing the motor to continually rotate.



(Click image to activate animation)

Most motors have a slightly more complex inner structure. Instead of a simple two-pole armature and two segment commutator, larger motors in particular often have a commutator divided into many segments. This allows the magnetic field to switch its orientation many times throughout a single rotation, optimizing the application of force to the armature.

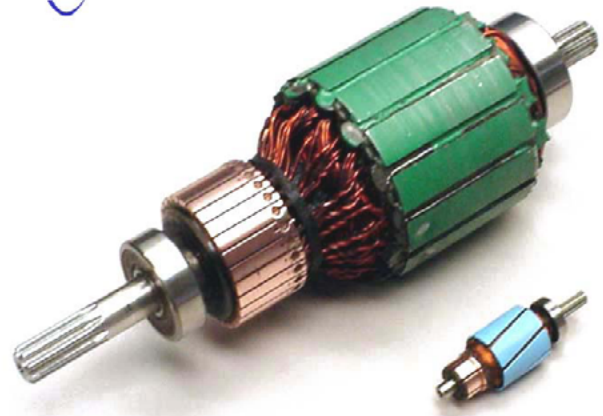


Figure 4: A few more realistic DC armatures, with multi-segment commutators. The brushes and permanent magnets are arranged such that the magnetic force is always nearly perpendicular to the armature lever arm.

7.3: Basic Concepts Behind Motor Curves

In section 7.1, we discussed the nature of motors as a power transducer. Now, we'll look more in detail at how to get the most out of a PMDC motor.

Let's start with a straightforward concept. As the load which a motor must lift increases, the motor will slow down. Eventually, the load could reach such a high level that the motor completely stops, or stalls. However, the motor will struggle against this condition, and output the most torque it possibly can to attempt to break out of the stall. This maximum torque value is known as stall torque.

What happens on the other end of the spectrum, if the motor is allowed to free spin? This time, speed will be at its maximum. But torque, far from being constant, is effectively zero. The motor only needs to output enough torque to overcome wind resistance and internal friction, and as a result, no usable torque is produced at the shaft. In between, we will see the same pattern occur, speed being reduced as torque increases, and vice versa. Up until it reaches stall, the motor will only output enough torque to overcome external forces and rotate at a **constant speed**.

This relationship between torque and speed can be graphed. Since the external torque a motor will have to move is typically known, torque is usually used as the independent variable, with speed on the Y axis as the dependent variable. A typical torque-speed graph can be found in Figure 5. In a DC motor, the relationship is always linear. Other types of motors have a more complex relationship between output torque and output speed.

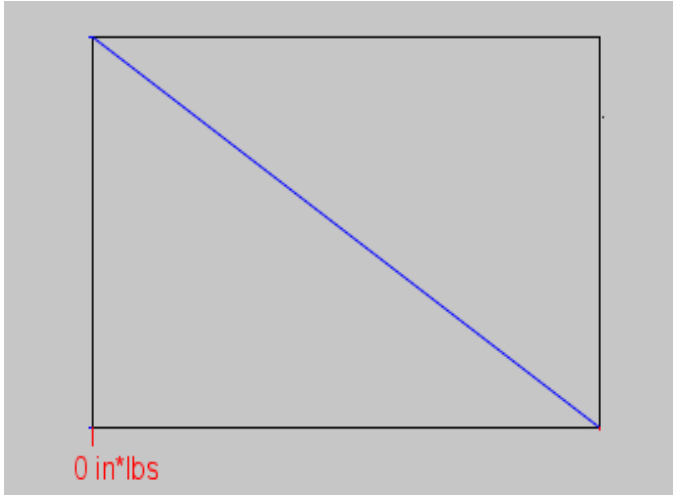


Figure 5: The inversely proportional relationship between speed and torque. The far left of the graph is free speed, the far right is stall

Other properties of electric motors discussed in section 7.1 vary with torque as well. A graph showing the speed, power, current draw, and efficiency of a motor as they relate to a changing output torque is known as a motor curve. An entire motor curve can be derived through four data points: **free speed, free speed, current draw, stall torque, and stall current draw.**

Based on your knowledge of mechanical power, transmissions, and the effect that increased load has on torque, try playing with the tool found [here](#). Then, read about the details of motor curves, for further insight on the topic.

7.4: Building a Motor Curve

Using this data, we can set up our motor curve (shown in [figure 6](#)). Start with the X axis for torque, which will go from zero (at free speed) to the measured stall torque. Speed can be graphed next, a downwards linear curve moving from the measured free speed on the left, to zero at stall on the right.

Figure 6: Complete motor curve. Click the frames below to isolate each part of the curve.

Current also shares a linear relationship with torque. The current measured at free speed should have been close to, but not quite, zero. This reflects the small amount of power required to overcome internal resistance of the motor. A much larger value should have been seen at stall. Draw a straight line between these two values.

Power comes next, and as discussed in [section 7.1](#), is the product of torque, speed, and a scaling factor. Since torque is zero on one end of the graph, and speed is zero on the other, power will also approach zero at each end. Multiplying the functions for torque and speed together will create an inverted parabola, with a peak at exactly 50% torque.

Finally, efficiency can be graphed. It is calculated using the formula $\text{Power}_{\text{OUT}}/\text{Power}_{\text{IN}}$. Plugging in the formula $P=IV$ for power in, multiplying the linear equation for current by the motor's voltage, and using the previously derived mechanical power formula, efficiency's curve can be graphed. It will typically appear as a skewed curve, with a peak at roughly 25% torque.

7.5: Interpreting a Motor Curve

A motor curve has four critical points on it: Free speed, peak efficiency, peak power, and stall. As has been discussed, free speed and stall are not particularly useful. Even though a motor is physically capable of putting out its free speed, it doesn't do you any good, because there is no torque, and therefore no power being produced there. And don't count on a motor to lift a load at its stall torque, because it will do so with zero speed. Stall is actually a particularly dangerous position, because even though the motor is operating at 0% efficiency, it is drawing a tremendous amount of current, and therefore, using a huge amount of electrical power. Besides wasting power, this can be dangerous, because this power has to go somewhere. It is burned up in the form of heat, which can easily damage a motor. This is especially hazardous to smaller air cooled motors, which can smoke within seconds of being stalled.

Motors are often designed to operate near either peak power or peak efficiency. At peak efficiency, the greatest percentage of electrical power input to the motor is being converted into usable mechanical power. This is useful in an application where sustaining battery power is a high priority. At peak power, the motor is doing the most mechanical work it is capable of. If a robot is required to lift a specified load, operating the motor at peak power will lift this load the fastest.

Any level of power below peak power can be achieved in two ways: with high speed and low torque, or low speed and high torque. Generally, it is preferable to stay on the high speed, low torque side of the motor curve. This is because of efficiency, and it's skewed curve. Operating on this side of the curve will generally give you higher efficiency, and will always draw less current. And drawing less current reduces the likelihood of damaging your motor

7.6: Using Transmissions to Choose Where a Motor Operates

There was a lot of talk in the last section about choosing at what torque to operate a motor. How is this done?

The first, and most obvious way to do this, is by changing the load the motor lifts. Say you have a 1 foot long pivoting arm, that needs to lift 5 pounds. But your motor can only output four foot*pounds, even at stall. Reducing the weight being lifted down to two pounds would work, but this isn't always possible.

The better way to alter the torque that a motor "feels" is through using transmissions. Transmissions are another type of transducer, which alter the torque and speed characteristics of a device. Transmissions are represented using ratios. For example, a transmission with a 2.5:1 ratio will multiply the torque output by a motor by 2.5, while dividing the speed it outputs by 2.5. This means that a motor that previously output 2 foot*pounds of torque at peak power, now can output 5 foot*pounds, while still performing at peak power.

In order to determine the transmission ratio you need to use, use this formula:

$$\text{Ratio} = \frac{\text{Desired Torque on Motor Curve} * \text{Transmission Efficiency}}{\text{Required torque}}$$

Included in Figure 7 is a tool which should help you get the feel for how transmissions affect a motor, and visualize how a motor behaves at different points on the motor curve. Experiment with different ratios and weights to develop an understanding of how they change the system's behavior.

This tool will also give you a targeted wattage to aim for. This is a typical design problem in many systems; the total mechanical wattage needed will be known, and the designer must set up the system in order to generate the correct amount of watts from the motor. If the desired wattage is below the motor's peak power, a larger motor must be used.

7.7: Formulae Used

$$\text{Motor Power} = T * \omega * \text{Scale Factor}$$

$$\text{Electrical Power} = I * V$$

$$\text{Motor Efficiency} = \text{Power}_{\text{OUT}} / \text{Power}_{\text{IN}}$$

$$\text{Ratio} = \frac{\text{Desired Torque on Motor Curve} * \text{Transmission efficiency}}{\text{Required torque}}$$

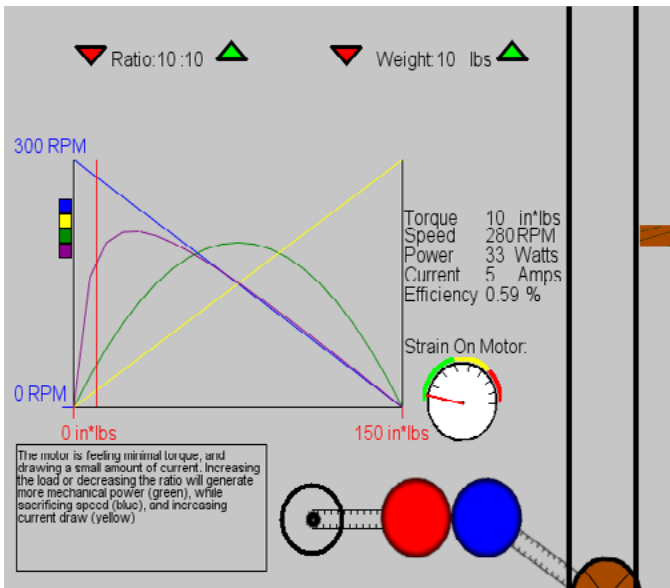


Figure 7: Click to activate the external motor curve simulation.

7.8: Concept Check Questions

1. A battery powered handheld vacuum gets clogged with a piece of paper. The motor can be heard to suddenly increase in speed. How does this affect the motor's current draw?

Increase

Decrease

No Change

2. Changing a motor's voltage also affects performance. Cutting a motor's voltage in half will also reduce the motor's stall torque and free speed by half. If the motor's original peak power was P, what is its new peak power?